Short communication

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THE SIGNED ALGORITHM AND ITS BUGS

ABSTRACT. Deaf children consistently lag behind their hearing cohorts in mathematics achievement tests. It has been hypothesized that their difficulty is a consequence of their lack of covert counting strategies and reliance on memorized verbal facts. We investigated the acquisition of an alternative method to solve sums, the signed algorithm, by six profoundly deaf primary school children. Similarly to the acquisition of the written algorithm by hearing children, deaf children's calculation errors with the signed algorithm were found to be systematic and related to the structure of the numeration system in British Sign Language. These results can be used to examine better ways of teaching arithmetic to deaf children and illustrate in a novel way the role of systems of signs in mathematical cognition.

1. The signed algorithm and its bugs

Research has consistently shown that deaf children lag substantially behind hearing children of the same age in mathematics achievement tests (National Council of Teachers of the Deaf, 1957; Wollman, 1965; Wood et al., 1983; Wood et al., 1984). It has been hypothesized that the reason for this delay is to be found in the way deaf subjects process numerical information. Hitch, et al. (1983), for example, hypothesized that deaf children's poor performance in arithmetic might be explained by their lack of subvocal, covert counting strategies which mediate hearing children's better performance in arithmetic tasks. Lacking these strategies, deaf children would simply have to rely on memory of addition and subtraction facts. However, deaf subjects' memory for digits is known to be significantly poorer than that of hearing subjects (Epstein et al., 1994). Thus reliance upon memory would be a poor strategy for deaf children to process numerical information.

What alternatives are there then for the teaching of deaf children? We examine in this study the acquisition and difficulties of a method of finding the solution for simple addition and subtraction computations which relies on signed numbers, termed here 'signed algorithm'. The reliance on signed numbers frees the children from the need to memorize verbally encoded addition and subtraction facts and provides them with an alternative to subvocal counting. This alternative is encouraging because it has been shown (Secada, 1984) that deaf users of American Sign Language matched

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to hearing counterparts in counting range are just as apt in using their counting system to answer simple numerical questions as hearing children. Therefore, a method for solving simple addition and subtraction sums based on signed numbers might significantly support deaf children's arithmetic learning in primary school.

Considering that there are no previous descriptions of this method, our aim was to describe the difficulties in mastering the signed algorithm when it is taught in the classroom by analyzing the bugs which emerge when children are using the method. The aim is akin to the analysis of bugs in written computation already described in the literature (see, for example, Brown and VanLehn, 1982; Resnick, 1982; Young and O'Shea, 1981). What we want to find out is whether children can learn the signed algorithm simply by copying a procedure, in which case their errors would be casual, miscounting errors, or whether there are difficulties in its acquisition which give origin to systematic errors. Knowledge of systematic errors can be used in the design of teaching strategies that takes their origin into account. It is not our aim to assess the impact of the signed algorithm on children's further development nor to compare it with other methods for teaching computation, a project which would require an experimental rather than the naturalistic approach used here.

1.1. *A brief note on the signed algorithm*

As far as we know, the signed algorithm is not a traditionally taught method although we have observed its use amongst a few deaf children in different schools. The cases reported here are of children who were systematically taught the signed algorithm. The teacher in this classroom had observed that one child used the method successfully; she learned it from the child and taught it to the other children. As far as we could ascertain, the child had not been taught the method and had come up with it himself. The interest of a method that appears to be spontaneously developed by children is obvious: the children might have found a psychologically sensible approach for themselves which could benefit other deaf children.

The algorithm involves simultaneously signing each of the numbers in an addition or subtraction sum with a different hand (see Figure 1 for the signed numbers used in this school). For example, one hand signs 8 while the other signs 7 when the pupil wants to solve $8 + 7$. Increments of one are then added to 8 (the value to be operated on) at the same time as 7 (the value of the transformation) is progressively decreased by one. The result is achieved when the hand signing the transformation reaches 0; it will be read on the other hand, which works as the notepad. In a subtraction

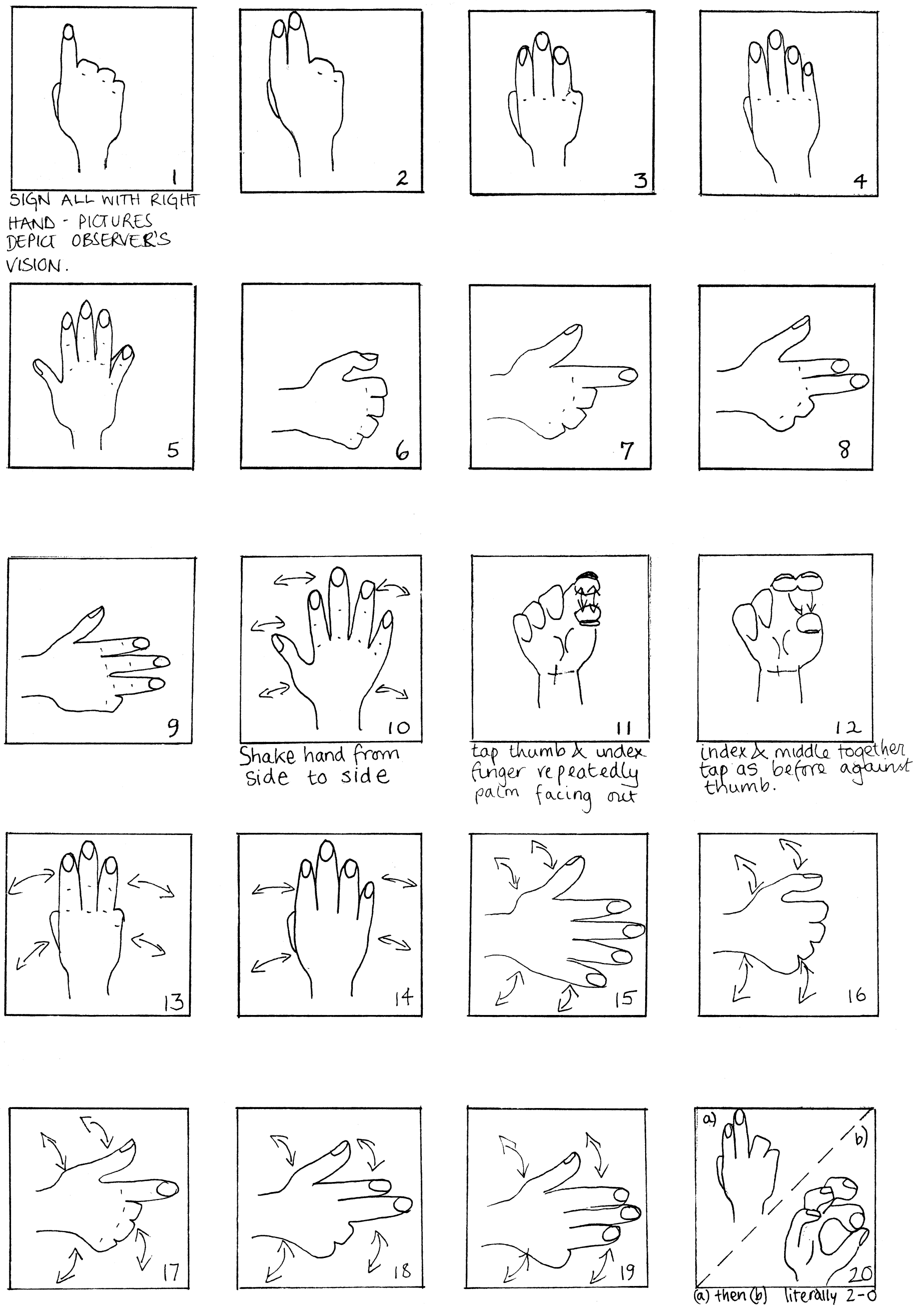
1
2
3
4
SIGN ALL WITH RIGHT HAND - PICTURES DEPICT OBSERVER'S VISION.
5
6
7
8
9
10
Shake hand from side to side
11
tap thumb & index finger repeatedly palm facing out
12
index & middle together tap as before against thumb.
13
14
15
16
17
18
19
a)
b)
20
(a) then (b) literally 2-0

Figure 1.

problem, the minuend is signed with one hand, the subtrahend with the other, and they are both decreased by one until the subtrahend reaches 0.

The use of the signed algorithm is quite distinct from the use of fingers by hearing children. When hearing children use their fingers to solve a sum, their fingers are (representations of) objects to be counted, they are not conventional signs. Any three fingers, for example, can represent three objects. In solving a problem, the children do not need to know which sum should be carried out: they can count the represented objects and model the actions in a story problem. It is possible to count on the fingers and solve, for example, a missing addend problem without knowing which sum could be used to find the answer (Marton and Neumann, 1990). In contrast, when deaf children use their fingers in the signed algorithm, the fingers represent numbers rather than objects: ‘three’ can only be signed in a particular, conventional manner and it is not true that any three fingers will do. Fingers are here conventional signs, not objects to be counted, and the operation to be carried out must be decided by the child in order for a problem to be solved.

Counting up and down to implement the algorithm also differs from simply ‘adding’ or ‘taking away’ fingers in hearing children’s use of fingers. Whereas it does not matter which fingers added or taken away when a hearing child counts on fingers, this does matter in the signed algorithm: Adding or taking away fingers in the wrong order might result in a hand configuration that is not a signed number or that means a different number (note, for example, that 3 and 8 involve three fingers but different ones).

2. Method

2.1. *Participants:*

We observed six profoundly deaf primary school children (age range 6 to 8 years; mean age 7.23), users of Sign Supported English, attending a London primary school for the deaf. All children were profoundly deaf; none had other disabilities and only one child had deaf parents.

2.2. *Design:*

The children were video-taped in the classroom during six mathematics lessons taught by a specialist teacher of the deaf. The main aim of the lessons was to teach the signed algorithm; it was used simply to solve sums in five of the lessons and to solve application problems in the last one, when the children were asked to calculate the value of hypothetical purchases of objects displayed with price tags on a table. During each lesson, two children were taped, yielding a total of two sets of observation per child. These records were used to describe the difficulties of acquiring the signed algorithm. Subsequently, the children were interviewed by an experimenter, user of Sign Supported English, who asked the children to solve a sample of sums with values up to 20 presented on separate cards (e.g., $8 + 7$; $13 - 4$). All the records were used to obtain a description of the bugs in signed algorithm.

3. Results

The analysis of the video-tapes showed that children have to conquer a number of difficulties before this computation procedure is mastered. However, similarly to other computation algorithms, children become better at computation once they have mastered the process. We classified the errors into four types.

First, *the children need to distinguish fingers as countable objects from fingers as signs for numbers*. As pointed out earlier on, when the children are using the signed algorithm they need to start from a conventional signed number and count up or down as they carry out the sum. In the first lessons, some children did not seem to realize the completely conventional nature of the process: they just added or took away fingers and sometimes ended up with non-readable answers. These errors were observed early on in the learning process and had been eliminated by the testing session.

Second, *the children need to become experts in counting-down in signed numbers*, a necessary skill for both addition and subtraction in signed algorithm. Two types of count-down errors were observed.

a) Failure to ‘carry’ the 5 or the 10. Signed numbers work as a double base system: (1) the digits 6, 7, 8 and 9 are signed by extending 1, 2, 3 and 4 fingers, respectively: the change of base is indicated by a change in the orientation of the hand and a different choice of the extended fingers (see Figure 1); (2) numbers above ten include a sign for the tens, and a sign for the units, with a spatial displacement between the two (see authors’ note). If the children fail to ‘carry the five’, 6, 7, 8, and 9 are read as 1, 2, 3 and 4, respectively. Failure to ‘carry the ten’ results in confusing, for example, 14 and 4 or 13 and 3. Failure to ‘carry the five’ and ‘carry the ten’ were still observed in five of the six children during the testing session after the series of lessons.

b) ‘Skipping 5’ may be observed when the children count down from a number above 6. For a correct count-down, the thumb must be retracted twice in succession, once going from 6 to 5, and then again counting down from 5 to 4. This type of error was only observed in the most inexperienced child and even in her case it was not frequent.

A third difficulty of the algorithm relates to *the need to distinguish the number operated on (signed by the hand that works as the pad) from the value of the transformation being carried out (signed by the active hand).* If at any point during the calculation process the children forget which is which, they reverse the progressive transformations, obtaining the wrong answer. This error was still observed in two children, even if only occasionally, in the testing session after the series of lessons.

A fourth source of difficulty was related to a specific teaching choice made by the teacher. Her aim was to get the children to realise that it is more efficient to use the larger addend as the number to be operated on irrespective of where it appears in the sum – that is, to use the 'min-strategy' (Groen and Parkman, 1972). During the lessons, she inverted the order of the addends when the smaller one appeared first on the card, and represented it as the value of the transformation in the solution process (e.g., $3 + 8$ would be solved as $8 + 3$). This inversion in the order of addends requires that the children understand the commutativity of addition. Although the most skillful child in the group had no difficulty in changing the order of the addends, all five others made errors when the order of addends was changed by the teacher in the classroom; they ended up confusing the number operated on with the value of the transformation during calculation. In the testing session, only the most skilful child ever inverted the order of the addends when solving the sums.

4. Discussion and Conclusions

The signed algorithm cannot be learned by the simple copying of gestures. The errors observed in this study were systematic rather than casual, miscounting errors. Deaf children's errors with the signed algorithm can be related directly to the structure of the counting system and the algorithm used in the same way as the bugs in written computation have been related to place value understanding and the mechanics of the written algorithm. These results, therefore, illustrate in a novel way the impact of a system of signs on mathematical reasoning : they show how signed numbers influence deaf children's computation process.

The identification of systematic errors in the signed algorithm can be used in the design of teaching strategies by teachers of the deaf who may wish to show their pupils this computation method. It is possible that it could be an important means of allowing deaf children to develop a sense of control over their computational process. However, further research is needed to investigate whether the acquisition of this computational method can enhance deaf children's arithmetical knowledge in other ways.

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NOTES

There are two common ways of signing numbers in the teens: either by successive signs indicating one (ten) and then the number of units in a new location to the left or by signing the number of units while shaking the hand. There is also a common departure in signing combinations of numbers with tens and units: children sign the number of tens, a zero, and then the number of units. Teachers often treat this non-conventional signing as correct.

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