An Intervention Program for Promoting Deaf Pupils' Achievement in Mathematics

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Our past research identified two aspects of deaf children's functioning that places them at risk for underachievement in mathematics. The first is their reduced opportunities for incidental learning, and the second is their difficulty in making inferences involving time sequences. This article examines the effectiveness of an intervention program to promote deaf children's numeracy that was designed to deal with these two factors. The design involved a comparison of 23 deaf pupils participating in the project with a baseline group formed by 65 deaf pupils attending the same schools in the previous year. The project pupils were tested before and after the intervention on the NFER-Nelson Age Appropriate Mathematics Achievement Test. The intervention was delivered by the teachers during the time normally scheduled for mathematics lessons. The project pupils did not differ from the baseline group at pretest but performed significantly better at posttest. They also performed at posttest better than expected on the basis of their pretest scores, according to norms provided by the NFER-Nelson Age Appropriate Mathematics Test for assessing the progress of hearing pupils. We conclude that the program was effective in promoting deaf pupils' achievement in numeracy.

We describe in this article an intervention project designed to raise the achievement of deaf pupils in mathematics. It is well established that deaf pupils lag behind hearing pupils in mathematics. The National Council of Teachers of the Deaf (1957) in England carried out a study with a large sample of deaf pupils and reported that deaf pupils were on average 2.5 years behind in mathematics achievement tests. About a decade later, Wollman (1965) reported similar results in a survey that included a third of the pupils from 13 schools for the deaf in the United Kingdom. Wood, Wood, and Howarth (1983) found that no improvement in this situation could be documented two decades later: in their study deaf pupils were approximately 3.4 years behind in mathematics achievement when compared to their hearing counterparts. In the first section of this article, we argue that it is possible to raise the mathematics achievement of deaf pupils. In the second section, we describe the intervention program we carried out and the results of its assessment. In the last section, we discuss the implications for future research.

Is It Possible to Raise the Mathematics Achievement of Deaf Pupils?

We (Nunes & Moreno, 1998) have argued that hearing loss cannot be treated as a cause of difficulties in mathematics but as a risk factor. Several findings in the literature suggest that hearing loss is not a direct cause of difficulties in mathematics. First, not all deaf pupils are weaker in math than their hearing counterparts: approximately 15% of the profoundly deaf pupils perform at average or above average levels in standardized tests (Wood et al., 1983). If hearing loss were a direct cause of difficulties in mathematics, there should be no

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deaf pupils displaying achievements adequate for their age level.

Second, most studies have found either no correlation or only a very small correlation (Nunes & Moreno, 1998; Wood et al., 1983) between the level of hearing loss and mathematics attainment. This result suggests that hearing loss is not a direct cause of difficulties in mathematics.

Third, the development of deaf pupils' counting (Secada, 1984) and computation skills (Hitch, Arnold, & Philips, 1983) and of their problem-solving abilities (Nunes & Moreno, 1997) seems to follow the same pattern as that of hearing pupils, albeit at a slower pace. This suggests that there is no deviation but only a delay in deaf pupils' mathematics learning.

We proposed the alternative hypothesis that hearing loss places children at risk for difficulties in learning mathematics. Unlike a causal hypothesis, a risk factor hypothesis supposes that it is possible to prevent a risk factor from leading to negative outcomes if the necessary steps are taken.

Our past research has identified two specific difficulties of deaf pupils that can explain, at least in part, why they are at risk for low achievement in mathematics.

First, deaf children have fewer opportunities for incidental learning as a consequence of their hearing loss. This hypothesis has already been proposed by Furth (1966) and Rapin (1986), who suggested that deaf youngsters' poor results in reasoning tasks and educational assessments can be explained by an "information deprivation" (Rapin, 1986, p. 214). Deaf youngsters lack access to many sources of information (e.g., radio, conversations around the dinner table), and their incidental learning may suffer from this lack of opportunity. Consequently, some concepts that hearing children learn incidentally in everyday life may have to be explicitly taught to deaf pupils in school. We (Nunes & Moreno, 1998) identified one mathematical concept—*additive composition*—crucial to progress in mathematics, that is often mastered by children before they enter school or quite early on in their school lives, and that seems to create a significant obstacle for deaf children. Additive composition refers to the understanding that any number can be seen as the sum of other numbers (Nunes & Bryant, 1996; Resnick & Omanson, 1987). There is evidence that hearing children learn about additive composition informally, probably through their experiences with money, among other things. If a child is asked to pay for a sweet that costs 8 cents, for example, a child who understands additive composition has no difficulty in using one nickel and three pennies to pay. About 60% of 6-year-old and virtually all 7-year-old hearing children succeed in this task (Nunes & Bryant, 1996). In contrast, many deaf children, including some as old as 10 and 11 years (Nunes & Moreno, 1998), cannot combine coins of different values into a single amount.

A second difficulty of deaf children, often reported by parents (see Gregory, 1995), is related to communication about time. Research has shown (Moreno, 2000) that deaf children have significantly more difficulty than hearing children in tasks that require them to process a sequence of events over time, keeping in mind a gap for an unknown element in the sequence. Moreno's task required the children to listen to a short description of a sequence of events—for example, "there were some people waiting at the bus stop; the first person who got to the bus stop was a girl; the second one, I don't remember; the third one was a lady." The child was asked to show which one of four pictures could be the correct picture for this story. Only one alternative had a girl in the first position and a lady in the third position in the bus line. Moreno also used a control task, where all the three elements in the sequence in the story were mentioned and there was no need to think of holding an empty space in the line for a person not recalled by the experimenter. The deaf children's performance in the control tasks did not differ from that of hearing children of the same age. In contrast, the hearing children performed significantly better when there was a gap in the sequence. Moreno (2000) further showed in a short-term longitudinal study, by means of a series of regression analyses, that the deaf children's performance in this task was a strong predictor of their scores on a standardized mathematics test (the NFER-Nelson) 7 months later, even after controlling for age and IQ. Thus, deaf children may need support when communicating and reasoning about time, particularly if they need to consider a gap in a story sequence.

The ability to think of time and consider gaps in a description is often required in the mathematics classroom when we teach pupils about the inverse relation between addition and subtraction. Consider the problem "Mary had some sweets; her grandmother gave her three and now she has eight. How many did she have to begin with?" If pupils have difficulty in making inferences in connection with time sequences, it will be very difficult for them to work with problems that require them to think about inversion and time. In a previous study (Nunes & Moreno, 1997), we showed that deaf pupils were not behind hearing pupils when they solved problems that did not involve inversion but were considerably behind when the problems required such inverse inferences. Later, Moreno found that deaf children's performance in tasks that require making inferences about events over time is significantly behind that of hearing pupils of the same age level and predicts their mathematics achievement scores after an interval of 7 months.

To promote the development of deaf pupils in mathematics, we developed a program with two aims. The first was to give the deaf pupils opportunities to learn core mathematical concepts that many hearing pupils may learn informally outside school and to promote connections between these informal concepts and mathematical representations used in school. The second aim was to promote deaf pupils' access to information about word problems related to transformations over time by representing the problems through drawings and diagrams and reducing the need to retain information about sequences of events in memory.

Method

Participants

The project involved two groups of deaf pupils: the baseline group and the project pupils, all drawn from the same six schools for the deaf or schools with units for deaf pupils in London, United Kingdom.

The baseline group consisted of 65 pupils who were attending the same schools as the project pupils and were tested on the NFER-Nelson age-appropriate mathematics test approximately 1 year before the beginning of the project. They were drawn from a larger sample of 82 pupils according to the criterion of being in the same year groups (2–5) as the project pupils and having taken the same age-appropriate NFER-Nelson tests.

Table 1 Number of pupils in the project and baseline groups by age-appropriate test taken at pre- and posttest

	7 years	8 years	9 years	10 years	11 years
Project pupils at pretest ($n = 23$)	10	4	5	4	0
Project pupils at posttest ($n = 23$)	3	8	3	7	2
Baseline pupils ($n = 65$)	22	19	19	5	0

Project pupils were 23 children in six different classes whose teachers agreed to implement the program and participate in the meetings with the researchers during the period of the project.[1] The distribution of pupils per age appropriate test taken is presented in Table 1.

Design

Before the implementation of the program, the project pupils were assessed in the NFER-Nelson age-appropriate mathematics tests (beginning autumn 1998). The program was administered by the teachers at their own pace, making use of some of the time (approximately 1 hour per week) normally scheduled for mathematics lessons over two terms (end of autumn 1998, spring and beginning summer 1999). The program did not take up the whole time dedicated to the teaching of mathematics so that the teachers continued with their regular teaching of the curriculum in the remainder of the available time. At the end of the program, the pupils were assessed again in the NFER age-appropriate mathematics test (autumn 1999). Table 1 shows that some of the children were tested on the same age-appropriate test at the end of the program and others had moved on in age to the following testing level; 17 of the 23 children took the next age-level test at the end of the program.

The baseline group was assessed only once (autumn–spring 1996). Thus, most of the project children are compared with different baseline groups at pre- and posttest because on each occasion they are compared with the children who took the same age-appropriate test.

Procedure

The program was designed with the involvement of nine teachers of the deaf who attended monthly meetings with the researchers over five school terms. In the spring and summer terms of 1998, the researchers prepared basic materials relevant to the teaching of each of the key concepts and discussed each set of materials with the teachers. The teachers tried out the materials and reported back on the results, positive features, and the difficulties that the pupils had experienced. The materials were revised on the basis of the teachers' feedback.

The revised program was organized in the booklets, which were used in the implementation of the program in the classroom (examples are presented in Figures 1 through 8). The pupil booklets contained pictures to support the presentation of the situations and no text. The teachers received a teacher book that contained the same pictures along with instructions explaining the question. Teachers were asked to give the instructions to their pupils in the pupils' language of instruction (British Sign Language [BSL], Sign-Supported English [SSE], or English). They were free to adapt the instructions to the pupils' language knowledge.

All teachers were asked to attempt all the items with the pupils, even if they expected some of the concepts to be either too easy or too difficult for their pupils. The teachers were encouraged to introduce new items with practical materials if they felt this was needed to ensure that the children understood the situations. They were also encouraged to use discussions among the pupils as the items were solved. They provided their own explanations to the pupils. The program booklets worked as a starting point for them to work on everyday mathematical concepts. The teachers were encouraged to use the program in their own way, as long as they gave the pupils all the tasks in the booklets and followed the instructional sequence. This makes the program more similar to an off-the-shelf book than to a recipe to be strictly followed by teachers in detail.

Observations and teachers' reports made it clear that there was much variation in the implementation. Some teachers promoted pupil interaction more than others. Some teachers allowed the children to identify mistakes themselves by comparing their work, whereas other teachers provided feedback themselves. Teachers of older children tended to use materials less often than those of younger children when introducing problems. The materials chosen were sometimes simply blocks to be used as representations of other objects described in the problems and sometimes cut-out shapes (for example, little shorts and shirts) that offered more figurative representations of the objects mentioned in the problems. In spite of these variations, all the teachers worked from the booklets and reported that the children enjoyed the work.

The program was not envisaged as a replacement to the mathematics curriculum taught in the classroom. Its aim was to bring the deaf children's informal mathematical understanding to a solid basis for learning the curriculum that they are taught in school.

The Program

The concepts covered in the program were:

1. additive composition and its application to number and measurement;
2. additive reasoning (that is, reasoning about the relations between addition and subtraction as inverse of each other);
3. multiplicative reasoning (that is, reasoning about the relations between multiplication and division); and
4. ratio and fractions.

Each concept was explored by means of a series of tasks. In each series, the items were ordered according to their expected level of difficulty. The most difficult items in one series were often more difficult than the easier items in the following series. This gave the children a sense of progress in one series and also the feeling that some easier tasks would be coming later on.

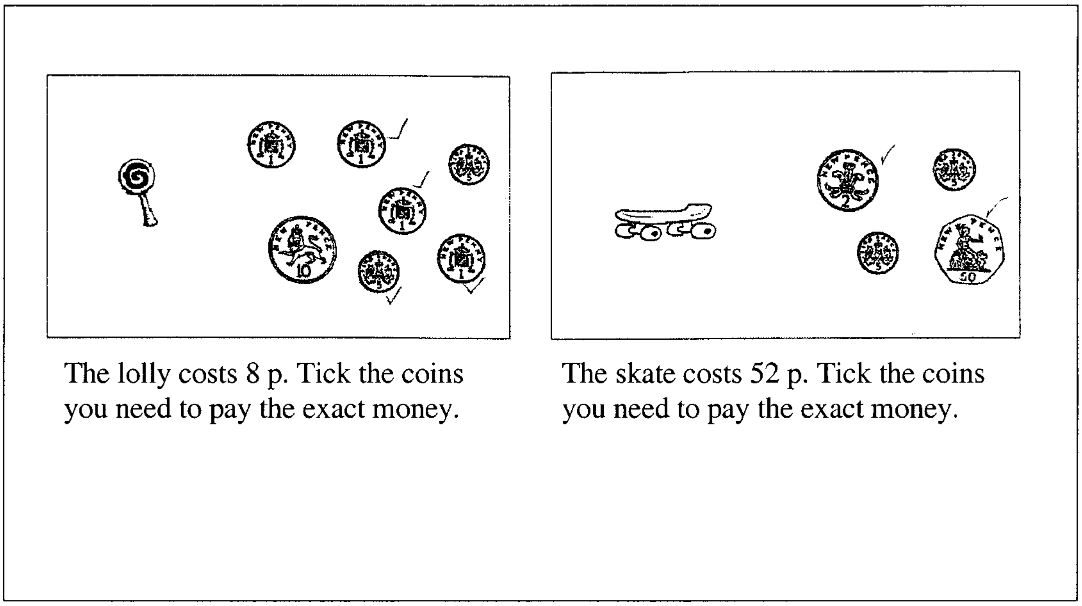


Figure 1 Two examples of additive composition items.

The work with each concept is briefly described in the following sections.

The aims of the section of the program on additive composition, number, and measurement were (1) to strengthen the pupils' understanding of additive composition; (2) to strengthen their understanding of how numbers are used to measure, thereby expanding the use of additive composition; and (3) to introduce the number line as a working tool for representing and solving problems.

Items on additive composition. Two examples are included in Figure 1. The aim of the items was to use pupils' informal knowledge of money and strengthen their understanding of additive composition. We discussed with the teachers the difficulties of each type of item and different ways of promoting children's understanding of the crucial concepts. For example, pupils who have difficulty with additive composition benefit from representing the value of nonunitary coins (nickel or dime) with their fingers before counting the total sum of money. They also benefit from working with values where the number words facilitate the task (e.g., combining a 20p coin with 1p coins) before working with values where the number words do not help (e.g., combining 10p and 1p coins). When pupils succeed in these simpler tasks, they can go on to more difficult items and be asked to think about how they solved the previous items.

Items on measurement. We saw two advantages of including items on measurement in this series of tasks related to number concepts. First, it is important that pupils think about the quantities represented by numbers in order to strengthen their understanding of number. Our previous work indicated that pupils' understanding of measurement is often incomplete. When measuring, many are not sure where to start measuring from, the edge of the ruler, zero, or one. This suggests that they do not fully understand what the reading obtained from a ruler indicates. Second, rulers can be used as a number line. Thus, they are useful in strengthening pupils' familiarity with mathematical conventions. Some of the items involved measurement with a broken ruler. Measuring with a broken ruler reinforces pupils' understanding that it is possible to work with number lines that start at any point. Some items from this section are presented in Figure 2.

Our discussions with the teachers revealed that even some of their older pupils were not aware of the

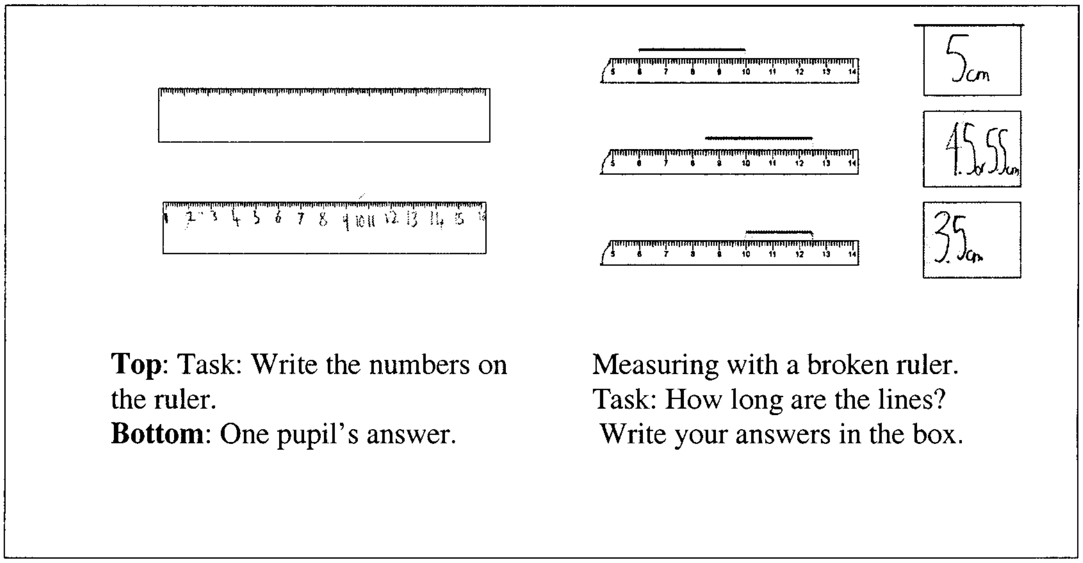


Figure 2 Two items from the measurement tasks.

precise meaning of the numbers obtained from reading a ruler. The examples in Figure 2 show how a pupil counts the "longer lines" on the ruler as an indication of the number of centimeters and the shorter lines as an indication of the number of half centimeters. The pupil counted the lines, not the units of length.

Items for introducing work with the number line. Work with the number line was included in the program for two reasons. First, it is a form of conventional mathematical representation. Second, the number line offers a visual representation of number sequences. It is known from previous work that deaf adults process information presented visually more efficiently than information presented orally. Thus, we expect the number line to be a useful tool for deaf children when they calculate or discuss numerical information in the classroom. Two examples of tasks used to introduce the number line are presented in Figure 3.

The teachers reported that the children did not find it difficult to use the number line to represent an answer. This facilitated the transition to using it as an instrument in problem solving.

The aims of the section on additive reasoning were (1) to promote the coordination of addition and subtraction as inverse of each other; (2) to work with drawings and diagrams, representing time through spatial relations; and (3) to use the number line for calculation and for the demonstration of different solutions to the same problems.

Problems involving addition and subtraction with different number meanings and different levels of complexity (invisible addends, start unknown, comparison) were included to provide the pupils with the opportunity to explore additive reasoning broadly. Teachers were encouraged to use the pupils' records for the discussion of different ways of solving the same problems. Some examples of items are presented in Figures 4 and 5.

The classroom observations showed that some teachers used concrete materials to help the younger children reason about problems that they found more difficult. Once the pupils had solved some problems using objects, they were able to work with the booklet without difficulty. The connection between the way in which they had counted the objects and counting on the number line was easily made.

Problems with start unknown (Mary had some sweets; her friend gave her two; now she had eight; how many did she have to begin with?) and with transformation unknown (A boy had five cakes; he ate some; now he has three; how many did he eat?) are quite

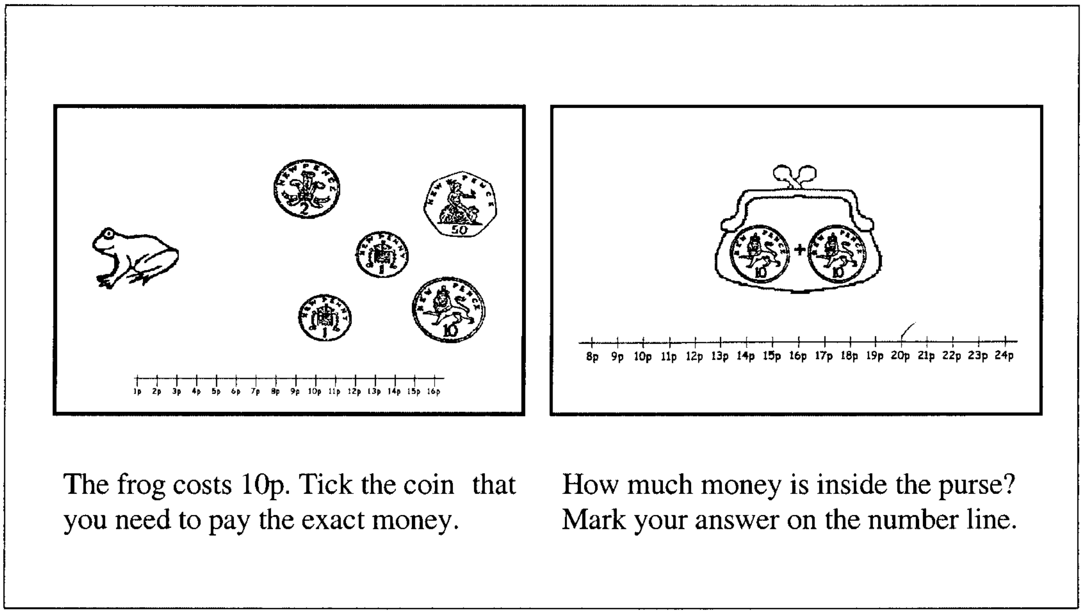


Figure 3 Two items introducing the number line.

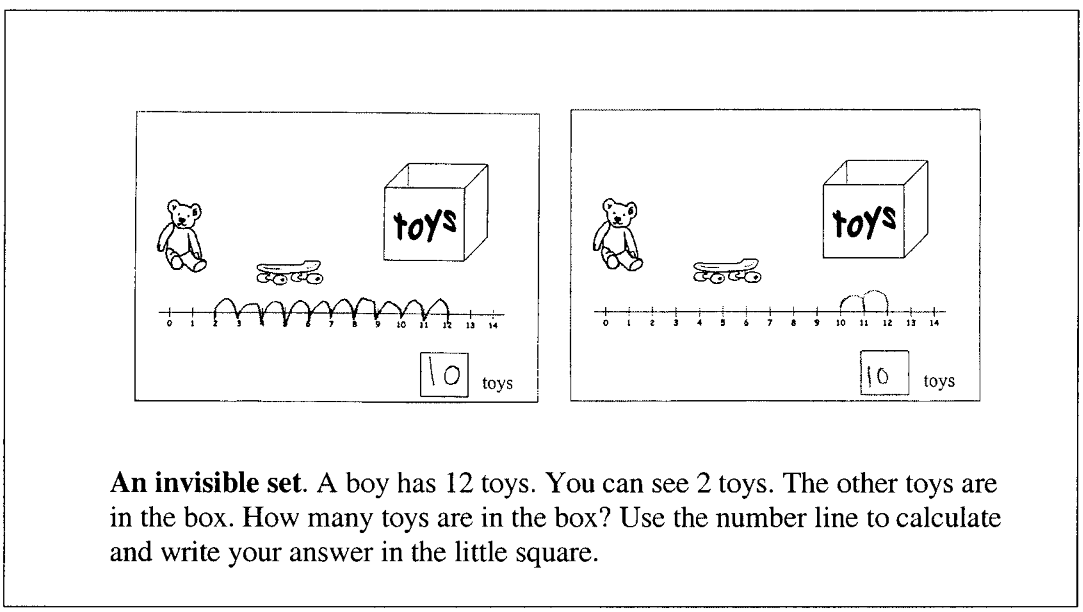


Figure 4 Two ways of reasoning leading to the same answer. A discussion between the pupils can show the co-ordination between addition and subtraction.

difficult for deaf pupils, as they involve making time-related inferences. One example is included in Figure 5.

Comparison problems are the most difficult of the additive problems with natural numbers for hearing and deaf pupils alike. Our program introduced comparison problems by initially connecting the comparison to additive transformations. Our previous research (Nunes & Bryant, 1996) has shown that this is an effective way to make the solution of comparison problems accessible to 6-year-olds. One example is shown in Figure 5. Several were included in the program. Teachers reported that the number line was a particu-

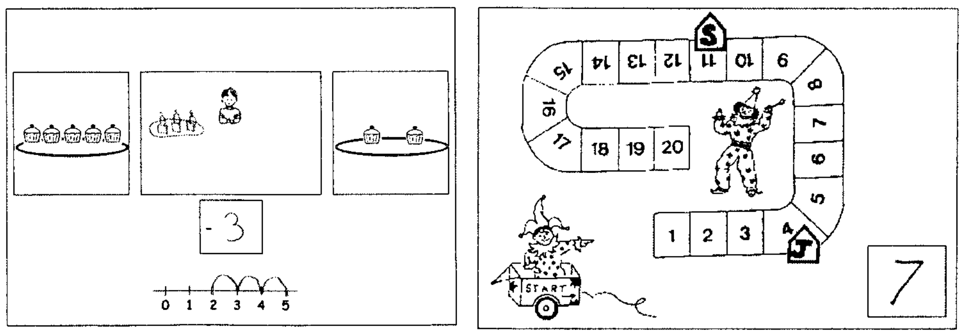


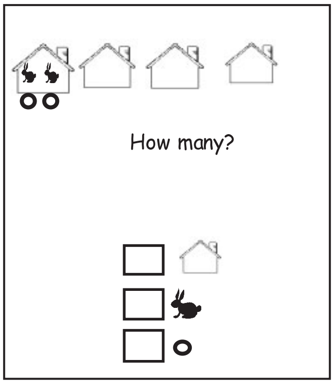
Figure 5 A problem with a missing transformation and a comparison problem.

larly useful instrument when discussing the logic of comparisons.

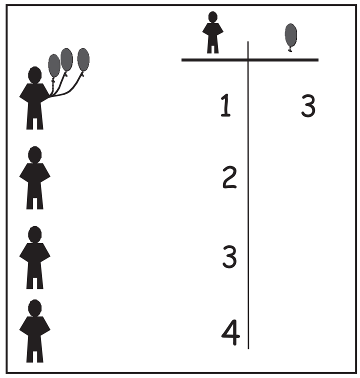
The aims of the section on multiplicative reasoning were (1) to work from pupils' intuitive understanding of correspondence as the basis of multiplicative reasoning; (2) to work with drawings and diagrams that represent two variables in the representation of multiplicative concepts; and (3) to introduce tables and graphs as mathematical representations for multiplicative relations. Some examples are presented in Figures 6 and 7.

Much research has shown that pupils' misconceptions in the domain of multiplicative reasoning are rooted in the idea that multiplication is simply repeated addition (for a review, see Greer, 1992). We (Park & Nunes, 2000) have carried out an intervention study with hearing children ages 6 and 7 years, comparing instruction on multiplication as repeated addition and as an operation linked to the schema of correspondence. Children instructed through the correspondence schema performed significantly better in the posttest than those instructed through repeated addition. Some examples of multiplicative reasoning problems are included in Figure 6. The two items on top exemplify the connection between the correspondence schema and the concept of multiplication. Tables are used as a way of representing the connection between the variables. The bottom items illustrate how the connection between multiplication and division was introduced. The text describing the problem situation was used to guide the teachers' explanation: each teacher was asked to present it to the pupils in the language used in the classroom, adapting or explaining the problem in a different way, without supplying further information. The problem on the bottom left describes a typical multiplication situation with a missing factor (number of children) and provides information about the product (total number of flowers). The pupils typically used a division strategy to solve this problem: they circled the flowers in groups of two: each group corresponds to one child in the class. The problem on the right presents an incomplete table where the ratio of flowers per vase has to be identified by division.

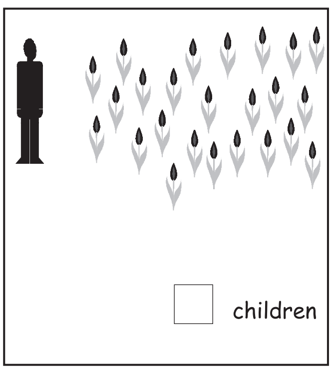
The items used to introduce graphs and their connection to multiplication were designed to help the pupils make a transition from more concrete drawings to more abstract diagrams. The graphs were always presented as representations of problems. In the initial problems, the pupils represented their answers in the



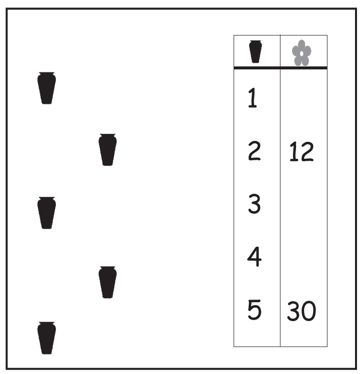
Two rabbits live in each house. We need one pellet of food for each rabbit. How many houses are there? How many rabbits? How many pellets of food do we need?



We are having a party. Each child that comes will get 3 balloons. Fill in the table to show how many balloons we need if 2, 3 and 4 children come to the party.



The teacher had birthday. Each child in the class brought him two flowers. The teacher received 24 flowers. How many children are in the class?



We are fixing the flowers for a party. Each vase will have the same number of flowers. Fill in the table with the number of flowers corresponding to the number of vases.

Figure 6 Four examples of multiplication and division items illustrating the use of tables.

graphs. Later, they were asked to obtain information from the graph. Two sample problems are presented in Figure 7.

An analysis of the pupils' productions showed that even the younger pupils were able to have some success in reasoning about and representing multiplicative problems with tables and graphs.

The aims of the section on fractions were (1) to work with sharing and division as an intuitive starting point for reasoning about fractions; (2) to promote a connection between pupils' understanding of fraction and division (¼ seen as one divided into 4), and between fraction and ratio (one out of four means the same as a 1:3 ratio). Figure 8 presents two examples, one of an item that connects sharing with fractions and the second that connects ratio and fraction.

The concept of fraction as traditionally taught (areas of a figure) was not stressed in the program. Although it is often used in the classroom, research suggests that children's intuitions about sharing and division can provide a solid start for understanding fractions from about age 8 (Streefland, 1997). Children

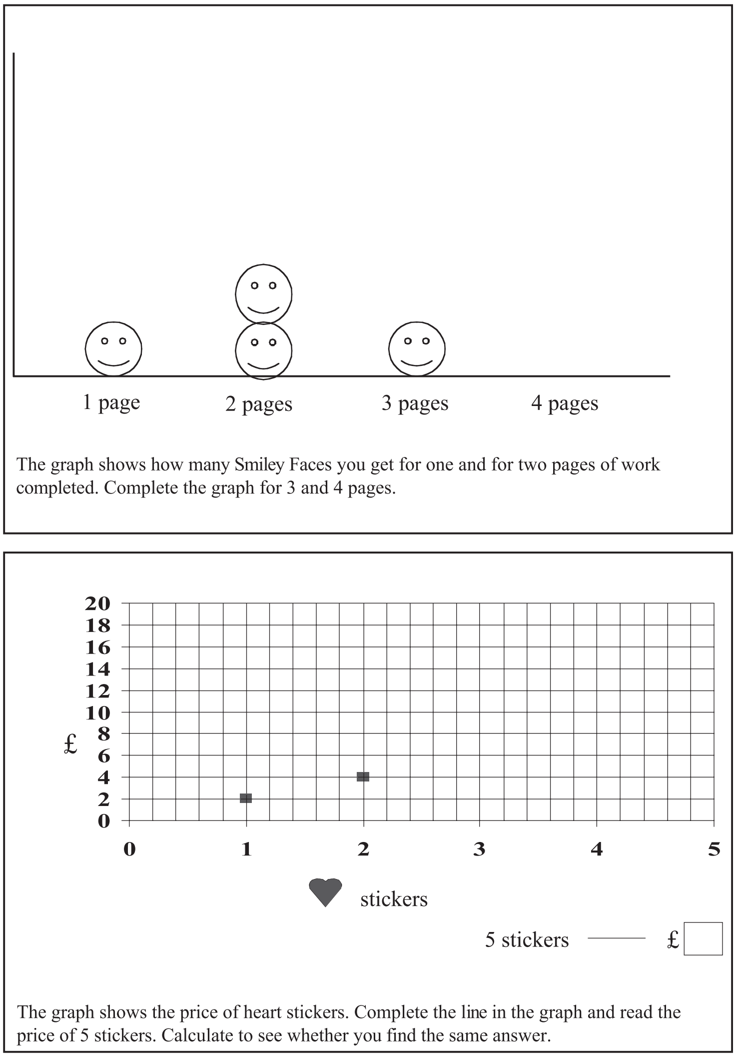


Figure 7 Two examples of items illustrating the introduction of graphs at the initial stages of the program (top) and the use of graphs at a later stage (bottom).

at this age already realize that one pie shared among two people will give larger pieces than one pie shared among four people. This helps them understand that ½ is a larger number than ⅓. A common misconception about fractions documented in the literature is to think that ⅓ is larger because 3 is more than 2.

The teachers found the fraction items difficult to work with, as they are not used to thinking about the connection between ratio and fraction. It should be pointed out, though, that the National Numeracy Strategy in England includes as a specific aim of teaching about fractions to establish a connection between the concepts of fraction and ratio. They also reported greater difficulty for the pupils but acknowledged that the easier items were accessible even to the youngest pupils.

Results

The effect of the intervention was assessed in two ways. The first analysis was a comparison between the scaled scores of the project pupils and those of the baseline group. Because different items and different numbers of items are used in the different age-level tests, a comparison that includes tests administered at different age levels must be based on scaled scores. Scaled scores take into account these variations and are obtained by

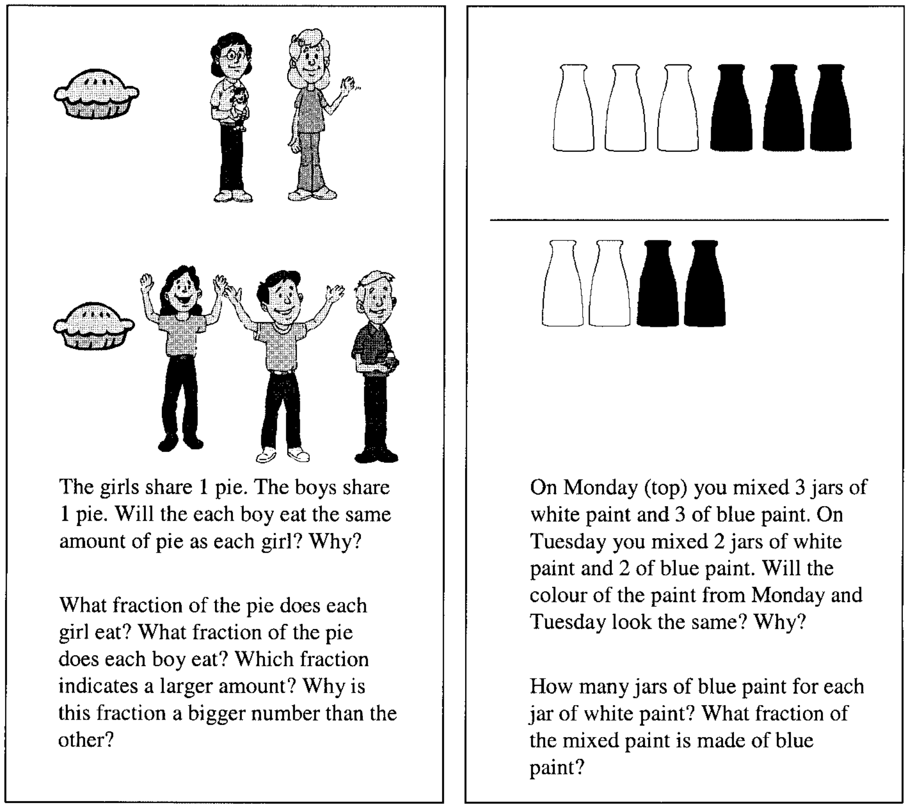


Figure 8 Two examples of problems dealing with ratio and fractions.

consulting a table for the appropriate test. Scaled scores in the NFER-Nelson vary between 0 and 76.

We carried out two comparisons, one between the pretest scores of the project pupils and the baseline group and the second between the posttest scores of the project pupils and the baseline. We had no reason to expect a significant difference between the project and the baseline pupils in the pretest because they had been drawn from the same schools. If the intervention were successful, the project pupils should perform significantly better than the baseline pupils in the posttest.

Figure 9 shows the mean scaled score for the project pupils at pre- and posttest and the baseline group for each age-appropriate test administered. The number of pupils for the different data points is specified in Table 1.

To test whether the differences observed were significant, we used two analyses of covariance with the level of test taken as the covariate, the group membership (project pupils vs. baseline group) as the independent variable, and the score in the pre- and in the posttest as the dependent variable, respectively in the first and second analysis.

The comparison between the project pupils and the baseline group did not produce significant results in the pretest: $F_{(1,84)} = .651$, ns. In contrast, the project pupils differed significantly from the baseline group at posttest, $F_{(1,85)} = 7.296$, $p = .008$. Thus, we conclude that the project pupils improved significantly in their mathematics achievement during the time they were engaged in the program.

The second analysis was a comparison between the project pupils' observed progress on the standardized mathematics assessment and their expected progress. If the project were successful, their performance in the posttest should be better than the performance expected from them on the basis of their pretest performance. The NFER norms for the age-appropriate mathematics tests include a prediction of performance on tests administered at a subsequent age, taking into

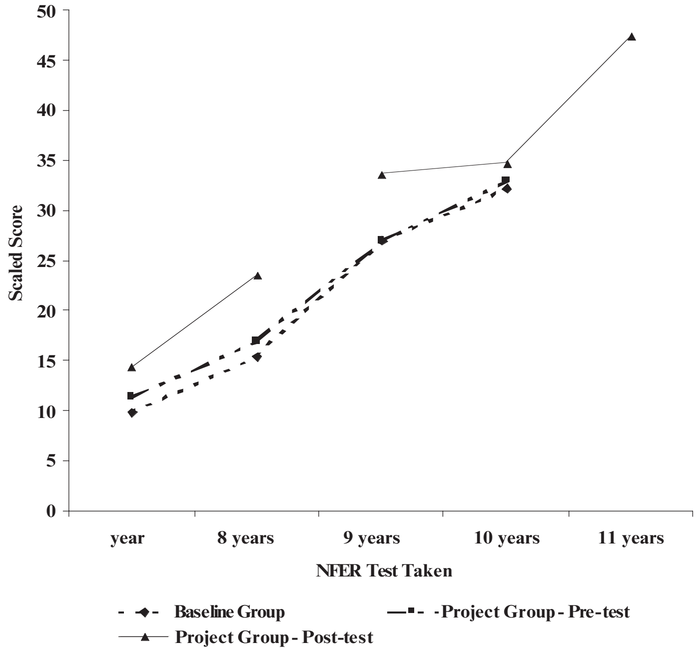


Figure 9 Mean scaled scores in the NFER-Nelson for the baseline group and the project group in the pre- and posttests by age-appropriate test taken.

account performance on a first testing occasion. These guidelines are produced for hearing rather than deaf children. Pupils whose observed performance is at a lower level than the predicted score are said to have made less progress than expected. Those pupils whose observed and predicted performances coincide are said to have made average progress. Finally, pupils whose observed score is superior to the predicted score are said to have made more progress than expected. By using the NFER guidelines, we obtained predicted scores for each pupil and compared these predicted scores to the observed scores. We observed that 31.8% of the pupils made less progress than predicted and 68.2% had higher observed than predicted scores. Because the distribution of the differences between observed and predicted scores was skewed, a nonparametric statistic, the Wilcoxon Signed Ranks Test for dependent samples, was used to compare the predicted with the observed scores. A significant difference indicates that the observed scores are significantly higher than the predicted scores. This analysis showed that the mean rank for the predicted scores was 7.86 and for the observed scores was 13.20 ($z = 2.31$, $p = .02$). The pupils' performance at posttest was thus significantly better than it would have been if their progress had been equivalent to the average amount of progress expected for hearing pupils during one school year.

Discussion

These quantitative analyses suggest that the intervention was effective in increasing the deaf pupils' access to the mathematics curriculum. The effects we documented cannot be explained solely in terms of the teaching the children would have normally received in the classroom because the baseline pupils had been exposed to the same curriculum in the same schools in the previous year. No major policy changes in the teaching of mathematics were introduced during this year in the schools.[2] The teachers did not increase the amount of time dedicated to teaching mathematics because the project was implemented during the time normally scheduled for mathematics lessons. No extra classroom helpers were made available during the project. Thus, there is no other reason beyond their participation in the project to expect that the pupils would perform significantly better than the baseline group nor that they would make more progress than predicted for hearing children during the year.

As in any other major intervention, where pupils participate in a large number of activities, it is not possible to say exactly what in the program led to the significantly greater level of progress. The program was designed with deaf children's needs in mind: the need for visual support in communication in the mathematics classroom and the need for the systematic teaching of concepts that hearing children might learn informally. The activities we proposed were scrutinized by the participating teachers of the deaf, who made suggestions for the visual and linguistic presentation. The tasks were organized in blocks that tackled the same concept from different perspectives and thus created for the children the opportunity of thinking about the same concept in different ways. The teachers encouraged the children to use drawings and diagrams to explain their answers, but the children did not have to invent these diagrams for themselves: the program provided them with examples of drawings, diagrams, tables, and graphs that could be used as tools for thinking and communicating about mathematics. The program consisted exclusively of activities that involve reasoning: no time was dedicated to the teaching of algorithms. This was in line with the decision to create a conceptual basis that would provide access to the mathematics curriculum instead of replacing it. All of these aspects of the program were novel when compared to the mathematics instruction at the time. Mathematics lessons, even for deaf pupils, seem to rely on language to a large degree. Calculations are taught through the recall of verbally represented number bonds and rules for how to proceed. When there are misunderstandings in the teacher's explanations, there are few resources to support a discussion of the mathematical ideas, which could show what had been lost in the communication. Visual means of representing relations between variables, such as tables and graphs, are typically not introduced in the context of problem solving, as was the case in this intervention; at this age level, the curriculum includes only bar graphs and the pupils are asked to read frequencies, without a consideration of other uses of graphs than the display of information. It is likely that all these differences jointly contributed to the positive impact of the program. It is not possible to tease out the effects of the different aspects of the experience provided by the program. Although the question of specific effects should be addressed in future research, it would have been outside the aims of this intervention, which were to maximize the learning opportunities for the deaf pupils. We believe that many factors change when a program such as this is introduced in the classroom and that experimental interventions outside the classroom are needed to separate the effects of different aspects of an educational program.

Likely, in this case, the effects should be attributed both to cognitive and motivational factors. The *cognitive effects* are likely to be based both on the specific design for teaching the core concepts included in the program and on the use of drawings and diagrams in teaching. The instruction program was carefully designed to use children's mathematical intuitions and to help them confront conceptual difficulties identified in previous research. The program also provided the pupils with tools for representing the core concepts in a mathematically adequate form. The use of drawings and diagrams was chosen for its potential in addressing the communication needs of deaf pupils. The pupils seem to have found drawings and diagrams useful as a means of representing their ideas and working toward solutions. The teachers reported that their pupils, after starting to work with the project booklets, had begun to use drawings and diagrams at other moments in the mathematics lessons.

Likely there were also *motivational effects* operating beyond these cognitive factors. Our own observations and the teachers' reports indicate that the pupils enjoyed working with the booklets. In one class they actually celebrated the moments when the teacher asked them to bring out the booklets: they clapped and showed great enthusiasm verbally. In another class the pupils did not want to interrupt their work at lunch time because they were too engaged in a discussion about measuring with a broken ruler, an event very unusual in the teacher's experience of this group of pupils.

Further research analyzing specific aspects of this intervention is necessary to identify the most crucial cognitive elements in the project. It would be important to know whether the teaching of concepts normally learned informally by hearing children is necessary or whether the use of visual means of communication alone can produce such positive results. If the teaching of informal concepts is necessary, and these have already been mastered by hearing children, schools would have to consider the need to offer extra teaching to the deaf children outside regular lessons. If the use of visual means of representations is sufficient to accomplish the desired results, these could well be introduced in the regular classroom where deaf and hearing children might be working together. Many hearing children likely would benefit from this more visual instruction too.

Notes

1. One of the teachers went on maternity leave during the project. The second author replaced her during this test, implementing the program 1 hour per week during the period scheduled for mathematics lessons.

2. After the conclusion of this project, English schools have started to implement a program known as The Numeracy Hour. There was no overlap between the implementation of the intervention described here and the Numeracy Hour.

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